Calculus 3 July 7, 2016

1. Find the image of the square $S = \{(u, v) \mid 0 \leq u \leq 1, 0 \leq v \leq 1\}$ under the transformation

$$x = u^2 - v^2 \qquad y = 2uv$$

2. Evaluate $\iint_R x+ydA$ where R is the trapezoidal region with vertices given by $(0,0), (5,0), (5/2, 5/2)$ and $(5/2, -5/2)$ using the transformation $x = 2u + 3v$ and $y = 2u - 3v$. Sketch the regions transformation first.

3. Evaluate $\iint_R ydA$, where R is the region bounded by the $x-$axis and the parabolas $y^2 = 4-4x$ and $y^2 = 4 + 4x$, and $y \geq 0$

4. Verify the dV for both cylinderical and spherical coordinates.

5. An standard ellipsoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Using what you know create "ellipsoidic" coordinates for a specific a,b and c. Then verify the volume equation of an ellipsoid which is $V = \frac{4}{3}\pi abc$.

6. Evaluate $\iint_R (x+y)e^{x^2-y^2} dA$, where R is the rectangle enclosed by the lines $x-y = 0, x-y = 2, x+y = 0, x+y = 3$

7. A vector field in $\mathbb{R}^2$ is defined by $\mathbf{F}(x, y) = \langle -y, x\rangle$. Describe $\mathbf{F}$ by sketching some of the vectors.

8. Sketch the vector field in $\mathbb{R}^3$ given by $\mathbf{F}(x, y, z) = \langle 0, 0, z\rangle$

9. Find the gradient vector field of $f(x, y) = x^2 + y^2$. Plot the gradient vector field on the contour map of f.

10. Find the gradient vector field of $f(x, y) = x^2y - y^3$. Plot the gradient vector field on the contour map of f.